

**The Box Method of Factoring a Polynomial.**Example:  $10x^2 + 11x - 6$ 1<sup>st</sup> create a 2x2 box


2<sup>nd</sup>, in the top corner put the first term in the bottom right corner put the last term.

$10x^2$	
	$-6$

3<sup>rd</sup>, multiply these two terms together to get  $-60x^2$ . Find two factors of  $-60x^2$  that when added together they will give you the middle term  $11x$ . These are  $15x$  and  $-4x$ . Put these into the open boxes.

$10x^2$	$15x$
$-4x$	$-6$

4<sup>th</sup>, factor the terms in each row and in each column.

	$2x$	$3$
$5x$	$10x^2$	$15x$
$-2$	$-4x$	$-6$

5<sup>th</sup>, the sum of the factors for the columns and the sum of the factors for the rows are the polynomial's factors:  $(2x + 3)(5x - 2)$

**Factor  $3x^2 + 16x + 5$** **Step 1 Place the first and last term in the box**

Use the box model shown at the top of your activity worksheet to factor  $3x^2 + 16x + 5$ . Place the  $x^2$  term in the upper left square of the box. Place the constant term in the lower right square of the box.

**Step 2 List factors**

Find the product of the terms in the box. Write it in the space provided on your worksheet. Then list the factors of the product. Be sure to list the factors as the product of a number and  $x$ .

**Step 3 Choose factors**

Find the sum of the factors you found in step 2. Circle the factors that add up to the middle term of  $3x^2 + 16x + 5$ .

**Step 4 Place the factors in the box**

Place one of the factors you circled in step 3 in one of the empty squares. Place the other factor in the remaining empty square.

**Step 5 Find the greatest common factor**

Find the GCF of the 1<sup>st</sup> column. Put this value in box (a).

**Step 6 Use multiplication**

The product of boxes (a) and (c) must equal the value in the upper left-hand square. To find the value of (c) ask, "What do you multiply the value in box (a) by to get  $3x^2$ ?" Put your answer in box (c).

**Step 7 Fill in remain boxes**

Repeat the procedure in Step 6 to find the values for boxes (b) and (d).

**Step 8 write the factors**

The sum of boxes (a) and (b) form one of the factors. The sum of boxes (c) and (d) form the other. Write the factors of the quadratic on your worksheet.

**DRAW CONCLUSIONS** Use your observations to complete these exercises

1. Use the box model to factor  $3x^2 + 13x + 12$ . You may want to refer to the steps in Explore 1.

a.       b.

c.

d.


In Exercises 2-4, use the box method to find the factors of the quadratic.

2.  $3x^2 + 11x + 10$       3.  $4x^2 + 15x + 9$       4.  $2x^2 + 11x + 14$

### EXPLORE 2 Factor $4x^2 + 5x - 6$ using the box method

#### STEP 1 Place the first and last terms in the box

Use the box model shown at the bottom of your activity worksheet to factor  $4x^2 + 5x - 6$ . Place the  $x^2$  term in the upper left square of the box. Place the constant term in the lower right square of the box. Find the product of the terms in the box and write it on your worksheet.

#### STEP 2 List factors

When the product of the first and last terms is negative, one of the factors is and one of the factors is positive. For example,  $-13x^2$  can have  $-1x$  as factors and  $13x$  or  $1x$  and  $-13x$  as factors. List all possible factors of the product in the space provided on your worksheet.

#### STEP 3 Choose factors

Find the sum of the factors you found in Step 2. Circle the factors that add up to the middle term of  $4x^2 + 5x - 6$ .

**STEP 4 Place the factors in the box**

Place one of the factors you circled in Step 3 in one of the empty squares. Place the other factor in the remaining empty square.

**STEP 5 Find the GCF and use multiplication**

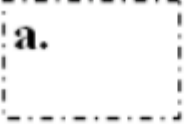
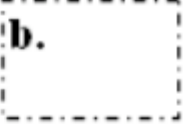
Find the GCF of the first column of the box. Put this value in box (a). Then use multiplication to find the values that go in boxes (b) - (d). Remember to record whether the numbers in boxes (b) - (d) are positive or negative.


**STEP 6 Write the factors**


The sum of boxes (a) and (b) form one of the factors of the quadratic. The sum of boxes (c) and (d) form the other. Write the factors of the quadratic on your worksheet.

**DRAW CONCLUSIONS** Use your observations to complete these exercises

5. Follow the steps in Explore 2 to complete the box model for  $3x^2 - 19x + 6$ .

**a.**  **b.** 

**c.** 


**d.** 

In Exercises 6-8, use the box method to find the factors of the quadratic.

6.  $5x^2 - 8x - 4$

7.  $6x^2 + 5x - 4$

8.  $x^2 - 2x + 1$

# Activity Worksheet

## EXPLORE 1

$$3x^2 + 16x + 5$$

**a.**

**b.**

Product: \_\_\_\_\_ Factors of the product:

**c.**

**d.**


$$3x^2 + 16x + 5 = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}})$$

## EXPLORE 2

$$4x^2 + 5x - 6$$

**a.****b.**

Product: \_\_\_\_\_ Factors of the product:

**c.****d.**


$$4x^2 + 5x - 6 =$$
$$(\quad)(\quad)$$



**EXPLORE 1 WORKED-OUT SOLUTION SHEET**

<p><b>STEP 1</b></p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="padding: 10px; text-align: center;"><math>3x^2</math></td> <td style="padding: 10px;"></td> </tr> <tr> <td style="padding: 10px;"></td> <td style="padding: 10px; text-align: center;">5</td> </tr> </table>	$3x^2$			5	<p><b>STEP 2</b></p> <p>Terms: <math>3x^2, 5</math></p> <p>Product: <math>3x^2 \cdot 5 = 15x^2</math></p> <p>Factors of <math>15x^2</math></p> <p><math>1x \cdot 15x</math></p> <p><math>3x \cdot 5x</math></p>												
$3x^2$																	
	5																
<p><b>STEP 3</b></p> <p>Sum of Factors: <math>1x+15x=16x</math> <math>3x+5x=8x</math></p> <p>Since the sum of <math>1x</math> and <math>15x</math> is equal to the middle term, <math>16x</math>, circle those factors.</p> <p>Factors of <math>15x^2</math> (<math>1x \cdot 15x</math>) <math>3x \cdot 5x</math></p>	<p><b>STEP 4 position of <math>1x</math> and <math>15x</math> may be switched</b></p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="padding: 10px; text-align: center;"><math>3x^2</math></td> <td style="padding: 10px; text-align: center;"><math>1x</math></td> </tr> <tr> <td style="padding: 10px; text-align: center;"><math>15x</math></td> <td style="padding: 10px; text-align: center;">5</td> </tr> </table>	$3x^2$	$1x$	$15x$	5												
$3x^2$	$1x$																
$15x$	5																
<p><b>STEP 5</b></p> <p>The GCF of <math>3x^2</math> and <math>15x</math> is <math>3x</math>.</p> <p>a. <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="padding: 5px;"><math>3x</math></td></tr></table>      b. <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="padding: 5px;"></td></tr></table></p> <p>c. <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="padding: 5px;"></td></tr></table></p> <p>d. <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="padding: 5px;"></td></tr></table></p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="padding: 10px; text-align: center;"><math>3x^2</math></td> <td style="padding: 10px; text-align: center;"><math>1x</math></td> </tr> <tr> <td style="padding: 10px; text-align: center;"><math>15x</math></td> <td style="padding: 10px; text-align: center;">5</td> </tr> </table>	$3x$				$3x^2$	$1x$	$15x$	5	<p>a. <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="padding: 5px;"><math>3x</math></td></tr></table>      b. <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="padding: 5px;"></td></tr></table></p> <p>c. <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="padding: 5px;"><math>x</math></td></tr></table></p> <p>d. <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="padding: 5px;"></td></tr></table></p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="padding: 10px; text-align: center;"><math>3x^2</math></td> <td style="padding: 10px; text-align: center;"><math>1x</math></td> </tr> <tr> <td style="padding: 10px; text-align: center;"><math>15x</math></td> <td style="padding: 10px; text-align: center;">5</td> </tr> </table>	$3x$		$x$		$3x^2$	$1x$	$15x$	5
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**STEP 7**a.  $3x$       b.  $1$ c.  $x$ 

	$3x^2$	$1x$
d. $5$	$15x$	$5$

**STEP 8**

$$3x^2 + 16x + 5 = (3x + 1)(x + 5)$$

**EXPLORE 2 WORKED-OUT SOLUTION SHEET****STEP 1**

$4x^2$	
	$-6$

**STEP 2**Terms:  $4x^2, -6$ Product:  $4x^2 \cdot -6 = -24x^2$ 

Because the product is negative, remember that one

factor will be negative and the other will be positive.

Factors of  $-24x^2$ 

$1x \cdot -24x \quad \text{or} \quad -1x \cdot 24x$

$2x \cdot -12x \quad \text{or} \quad -2x \cdot 12x$

$3x \cdot -8x \quad \text{or} \quad -3x \cdot 8x$

$4x \cdot -6x \quad \text{or} \quad -4x \cdot 6x$

**STEP 3**Since the sum of  $-3x$  and  $8x$  is equal to the middle term,  $5x$ , choose those factors.Factors of  $15x^2$ :  $1x \cdot -24x$  or  $-1x \cdot 24x$ 

$2x \cdot -12$  or  $-2x \cdot 12x$

$3x \cdot -8x$  or  $-3x \cdot 8x$

$4x \cdot -6x$  or  $-4x \cdot 6x$

**STEP 4 position of  $8x$  and  $-3x$  may be switched**

$4x^2$	$8x$
$-3x$	$-6$

## STEP 5

a.  $3x$ 

b. 4

c.

 $x$  $3x^2$  $4x$ 

d.

3

 $9x$ 

12

## STEP 6

$$4x^2 + 5x - 6 = (4x + (-3))(x + 2)$$
$$= (4x - 3)(x + 2)$$

**Answer Key****EXPLORE 1 AND 3**

See worked-out solution sheets.

**DRAW CONCLUSIONS**

1. Position of x-terms may vary.

<b>a.</b> $3x$	<b>b.</b> $4$				
<b>c.</b> $x$	<table border="1" style="border-collapse: collapse; width: 100%; height: 100%;"> <tr> <td style="width: 50%; text-align: center; padding: 10px;"><math>3x^2</math></td> <td style="width: 50%; text-align: center; padding: 10px;"><math>4x</math></td> </tr> <tr> <td style="width: 50%; text-align: center; padding: 10px;"><math>9x</math></td> <td style="width: 50%; text-align: center; padding: 10px;"><math>12</math></td> </tr> </table>	$3x^2$	$4x$	$9x$	$12$
$3x^2$	$4x$				
$9x$	$12$				
<b>d.</b> $3$					

2.  $(3x + 5)(x + 2)$
3.  $(x + 3)(4x + 3)$
4.  $(2x + 7)(x + 2)$
5. Position of x-terms may vary

<b>a.</b> $x$	<b>b.</b> $-6$				
<b>c.</b> $3x$	<table border="1" style="border-collapse: collapse; width: 100%; height: 100%;"> <tr> <td style="width: 50%; text-align: center; padding: 10px;"><math>3x^2</math></td> <td style="width: 50%; text-align: center; padding: 10px;"><math>-18x</math></td> </tr> <tr> <td style="width: 50%; text-align: center; padding: 10px;"><math>-1x</math></td> <td style="width: 50%; text-align: center; padding: 10px;"><math>6</math></td> </tr> </table>	$3x^2$	$-18x$	$-1x$	$6$
$3x^2$	$-18x$				
$-1x$	$6$				
<b>d.</b> $-1$					

6.  $(5x + 2)(x - 2)$
7.  $(2x - 1)(3x + 4)$
8.  $(x - 1)(x - 1)$

## Teacher Notes

### ACTIVITY PREPARATION AND MATERIALS

- Each student will need an activity worksheet. Students may need extra paper to write their factors.
- It will be helpful for students to have the worked-out solution sheets while doing Explores 1 and 2. Instruct the students to use the worked-out solutions to check their work at each step.
- Review multiplying binomials using the box method before beginning this activity. This will help students feel comfortable using multiplication to find the factors.

### ACTIVITY MANAGEMENT

- Some students will be able to mentally figure out which two factors add to get the middle term. Encourage all students to write out the factors initially, especially when working with negative numbers.
- Encourage students to check their answer using multiplication.
- **Common Error** Students need to be careful when factoring the product of the diagonal. Make sure they account for negative signs when necessary. Remind students that a negative times a negative is positive.
- **A-level Alternative** Work through the Explores as a class. Make sure you model the box method on the board for students. Fill in the box as you read through the steps. After completing Explore 1, you may want students to try Draw Conclusions Exercise 1 on their own or with a partner before continuing with Explore 2. After Explore 2, students can try Draw Conclusions Exercise 5.
- **C-level Alternative** Do not provide students with the worked-out solutions.

## Activity and Closure Questions

Ask these question as a class

1. Describe the signs of the factors of a quadratic that has a negative middle term and positive last term, such as  $x^2 - 5x + 6$ .

**Answer:** Sample answer: The factors of a quadratic with a negative middle term and a positive last term will both be negative. For example,  $x^2 - 5x + 6 = (x - 3)(x - 2)$ .

2. Describe the signs of the factors of a quadratic that has a negative middle term and a negative last term, such as  $x^2 - x - 6 = (x - 3)(x + 2)$

### LESSON TRANSITION

This activity provides students with a concrete method for factoring. You may want to do the examples in the book using the box method instead of doing the activity. If you do the activity, you may want to do Examples 4 and 5 before you assign homework.